A problem with the commonest ϵ convention for SL(2,C)

The purpose of this note is to mention a problem with the SL(2,C) ϵ convention commonly used in supersymmetry texts. The problem does not occur in Penrose's treatment of SL(2,C).

ϵ_{ab}, where a, b = 1, 2 is the antisymmetric tensor preserved by the action of SL(2,C) on account of the fact that it is unimodular. It is used to lower SL(2,C) indices and from this, ϵ^{ab} may be defined which can be used to raise indices.

The convention is as follows. Index raising is defined by

$$\psi^a = \epsilon^{ab}\psi_b \tag{1}$$

and index lowering is defined by

$$\psi_a = \epsilon_{ab}\psi^b \tag{2}$$

ϵ has the property that

$$\epsilon_{ab} = -\epsilon^{ab} \tag{3}$$

The index raising for a rank two tensor is thus

$$T^{ab} = \epsilon^{ac}\epsilon^{bd}T_{cd} \tag{4}$$

The problem with the convention is that for ϵ itself, index raising is performed differently, being

$$\epsilon^{ab} = -\epsilon^{ac}\epsilon^{bd}\epsilon_{cd} \tag{5}$$

which means that if one has, for example

$$T_{ab} = U_{ab} + \epsilon_{ab} \tag{6}$$

then it does not follow that

$$T^{ab} = U^{ab} + \epsilon^{ab} \tag{7}$$

in fact

$$T^{ab} = U^{ab} - \epsilon^{ab} \tag{8}$$

in other words, we have to remember to change the sign on ϵ when we raise or lower its indices. The Penrose conventions avoid this issue by requiring that (4) applies to ϵ at the outset, leading to a different rule for index lowering.

This is not a small problem: when I was a graduate student I spent days trying to track apparently-inexplicable signs appearing and disappearing in complicated calculations involving two-component spinors, only to trace the problem to this. If I had been aware of the need to change the sign of ϵ, then maybe things would have been alright. However, this behaviour is, to say the least, non-intuitive. Far better just to use Penrose's conventions and avoid the problem altogether.

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